16

Power and Harmonics in Nonsinusoidal Systems

Rectification used to be a much simpler topic. A textbook could cover the topic simply by discussing the various circuits, such as the peak-detection and inductor-input rectifiers, the phase-controlled bridge, polyphase transformer connections, and perhaps multiplier circuits. But recently, rectifiers have become much more sophisticated, and are now systems rather than mere circuits. They often include pulse-width modulated converters such as the boost converter, with control systems that regulate the ac input current waveform. So modern rectifier technology now incorporates many of the dc–dc converter fundamentals.

The reason for this is the undesirable ac line current harmonics, and low power factors, of conventional peak-detection and phase-controlled rectifiers. The adverse effects of power system harmonics are well recognized. These effects include: unsafe neutral current magnitudes in three-phase systems, heating and reduction of life in transformers and induction motors, degradation of system voltage waveforms, unsafe currents in power-factor-correction capacitors, and malfunctioning of certain power system protection elements. In a real sense, conventional rectifiers are harmonic polluters of the ac power distribution system. With the widespread deployment of electronic equipment in our society, rectifier harmonics have become a significant and measurable problem. Thus there is a need for *high-quality rectifiers,* which operate with high power factor, high efficiency, and reduced generation of harmonics. Several international standards now exist that specifically limit the magnitudes of harmonic currents, for both high-power equipment such as industrial motor drives, and low-power equipment such as electronic ballasts for fluorescent lamps and power supplies for office equipment.

This chapter treats the flow of energy in power systems containing nonsinusoidal waveforms. Average power, rms values, and power factor are expressed in terms of the Fourier series of the voltage and current waveforms. Harmonic currents in three-phase systems are discussed, and present-day standards are listed. The following chapters treat harmonics and harmonic mitigation in conventional line-commutated rectifiers, high-quality rectifier circuits and their models, and control of high-quality rectifiers.

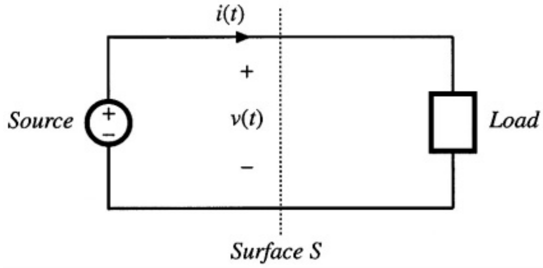

Fig. 16.1 Observe the transmission of energy through surface S.

16.1 AVERAGE POWER

Let us consider the transmission of energy from a source to a load, through a given surface as in Fig. 16.1. In the network of Fig. 16.1, the voltage waveform $v(t)$ (not necessarily sinusoidal) is given by the source, and the current waveform is determined by the response of the load. In the more general case in which the source output impedance is significant, then $v(t)$ and $i(t)$ both depend on the characteristics of the source and load. Balanced three-phase systems may be treated in the same manner, on a per-phase basis, using a line current and line-to-neutral voltage.

If $v(t)$ and $i(t)$ are periodic, then they may be expressed as Fourier series:

$$\begin{aligned} v(t) &= V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right) \\ i(t) &= I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right) \end{aligned} \tag{16.1}$$

where the period of the ac line voltage waveform is defined as $T = 2\pi/\omega$. In general, the instantaneous power $p(t) = v(t)i(t)$ can assume both positive and negative values at various points during the ac line cycle. Energy then flows in both directions between the source and load. It is of interest to determine the net energy transmitted to the load over one cycle, or

$$W_{cycle} = \int_0^T v(t)i(t)dt \tag{16.2}$$

This is directly related to the average power as follows:

$$P_{av} = \frac{W_{cycle}}{T} = \frac{1}{T}\int_0^T v(t)i(t)dt \tag{16.3}$$

Let us investigate the relationship between the harmonic content of the voltage and current waveforms, and the average power. Substitution of the Fourier series, Eq. (16.1), into Eq. (16.3) yields

$$P_{av} = \frac{1}{T}\int_0^T \left(V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)\right)dt \tag{16.4}$$

To evaluate this integral, we must multiply out the infinite series. It can be shown that the integrals of cross-product terms are zero, and the only contributions to the integral comes from the products of voltage and current harmonics of the same frequency:

$$\int_0^T \left(V_n \cos\left(n\omega t - \varphi_n\right)\right)\left(I_m \cos\left(m\omega t - \theta_m\right)\right)dt = \begin{cases} 0 & \text{if } n \neq m \\ \frac{V_n I_n}{2} \cos\left(\varphi_n - \theta_n\right) & \text{if } n = m \end{cases} \tag{16.5}$$

The average power is therefore

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos\left(\varphi_n - \theta_n\right) \tag{16.6}$$

So net energy is transmitted to the load only when the Fourier series of $v(t)$ and $i(t)$ contain terms at the same frequency. For example, if $v(t)$ and $i(t)$ both contain third harmonic, then net energy is transmitted at the third harmonic frequency, with average power equal to

$$\frac{V_3 I_3}{2} \cos\left(\varphi_3 - \theta_3\right) \tag{16.7}$$

Here, $V_3 I_3/2$ is equal to the rms volt-amperes of the third harmonic current and voltage. The $\cos(\phi_3 - \theta_3)$ term is a displacement term which accounts for the phase difference between the third harmonic voltage and current.

Some examples of power flow in systems containing harmonics are illustrated in Figs. 16.2 to 16.4. In example 1, Fig. 16.2, the voltage contains fundamental only, while the current contains third har-

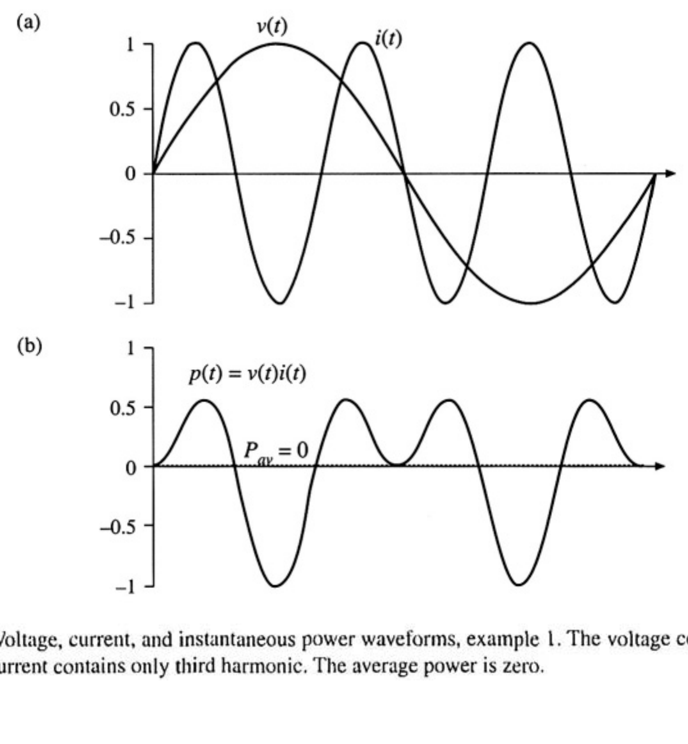

Fig. 16.2 Voltage, current, and instantaneous power waveforms, example 1. The voltage contains only fundamental, and the current contains only third harmonic. The average power is zero.

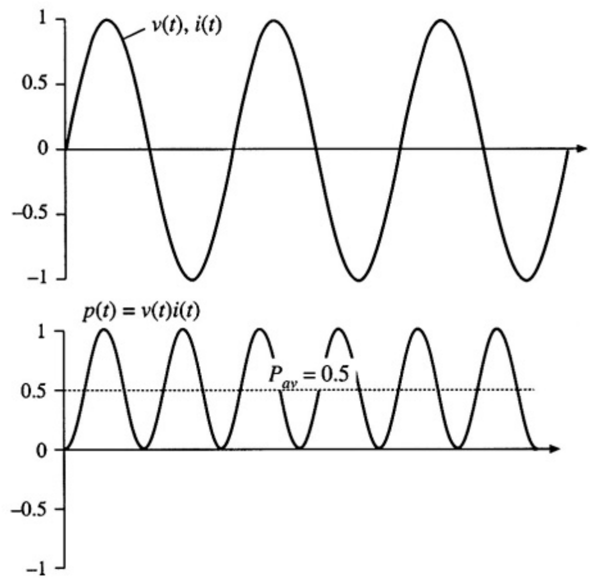

Fig. 16.3 Voltage, current, and instantaneous power waveforms, example 2. The voltage and current each contain only third harmonic, and are in phase. Net energy is transmitted at the third harmonic frequency.

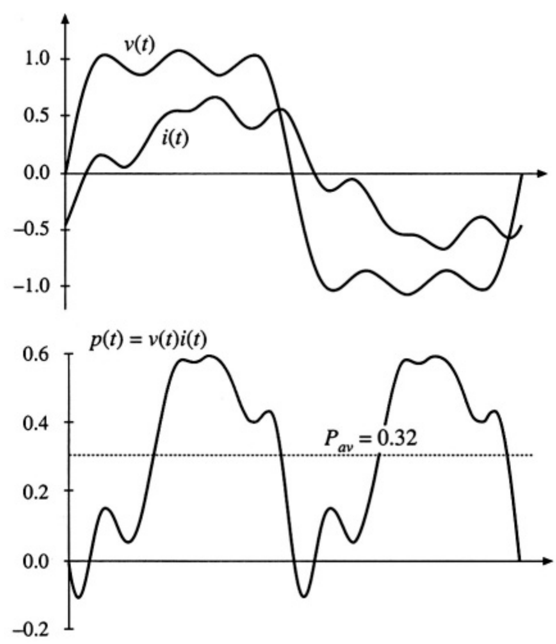

Fig. 16.4 Voltage, current, and instantaneous power waveforms, example 3. The voltage contains fundamental, third, and fifth harmonics. The current contains fundamental, fifth, and seventh harmonics. Net energy is transmitted at the fundamental and fifth harmonic frequencies.

monic only. It can be seen that the instantaneous power waveform $p(t)$ has a zero average value, and hence P_{av} is zero. Energy circulates between the source and load, but over one cycle the net energy transferred to the load is zero. In example 2, Fig. 16.3, the voltage and current each contain only third harmonic. The average power is given by Eq. (16.7) in this case.

In example 3, Fig. 16.4, the voltage waveform contains fundamental, third harmonic, and fifth harmonic, while the current contains fundamental, fifth harmonic, and seventh harmonic, as follows:

$$\begin{aligned} v(t) &= 1.2 \cos(\omega t) + 0.33 \cos(3\omega t) + 0.2 \cos(5\omega t) \\ i(t) &= 0.6 \cos(\omega t + 30°) + 0.1 \cos(5\omega t + 45°) + 0.1 \cos(7\omega t + 60°) \end{aligned} \tag{16.8}$$

Average power is transmitted at the fundamental and fifth harmonic frequencies, since only these frequencies are present in both waveforms. The average power is found by evaluation of Eq. (16.6); all terms are zero except for the fundamental and fifth harmonic terms, as follows:

$$P_{av} = \frac{(1.2)(0.6)}{2} \cos(30°) + \frac{(0.2)(0.1)}{2} \cos(45°) = 0.32 \tag{16.9}$$

The instantaneous power and its average are illustrated in Fig. 16.4(b).

16.2 ROOT-MEAN-SQUARE (RMS) VALUE OF A WAVEFORM

The rms value of a periodic waveform $v(t)$ with period T is defined as

$$(\text{rms value}) = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} \tag{16.10}$$

The rms value can also be expressed in terms of the Fourier components. Insertion of Eq. (16.1) into Eq. (16.10), and simplification using Eq. (16.5), yields

$$(\text{rms value}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}} \tag{16.11}$$

Again, the integrals of the cross-product terms are zero. This expression holds when the waveform is a current:

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}} \tag{16.12}$$

Thus, the presence of harmonics in a waveform always increases its rms value. In particular, in the case where the voltage $v(t)$ contains only fundamental while the current $i(t)$ contains harmonics, then the harmonics increase the rms value of the current while leaving the average power unchanged. This is undesirable, because the harmonics do not lead to net delivery of energy to the load, yet they increase the $I_{rms}{}^2R$ losses in the system.

In a practical system, series resistances always exist in the source, load, and/or transmission wires, which lead to unwanted power losses obeying the expression

$$(\text{rms current})^2 R_{series} \tag{16.13}$$

Examples of such loss elements are the resistance of ac generator windings, the resistance of the wire connecting the source and load, the resistance of transformer windings, and the resistance of semiconductor devices and magnetics windings in switching converters. Thus, it is desired to make the rms current as small as possible, while transferring the required amount of energy and average power to the load.

Shunt resistances usually also exist, which cause power loss according to the relation

$$\frac{(\text{rms voltage})^2}{R_{shunt}} \tag{16.14}$$

Examples include the core losses in transformers and ac generators, and switching converter transistor switching loss. Therefore, it is desired to also make the rms voltage as small as possible while transferring the required average power to the load.

16.3 POWER FACTOR

Power factor is a figure of merit that measures how effectively energy is transmitted between a source and load network. It is measured at a given surface as in Fig. 16.1, and is defined as

$$\text{power factor} = \frac{(\text{average power})}{(\text{rms voltage})\ (\text{rms current})} \tag{16.15}$$

The power factor always has a value between zero and one. The ideal case, unity power factor, occurs for a load that obeys Ohm's Law. In this case, the voltage and current waveforms have the same shape, contain the same harmonic spectrum, and are in phase. For a given average power throughput, the rms current and voltage are minimized at maximum (unity) power factor, that is, with a linear resistive load. In the case where the voltage contains no harmonics but the load is nonlinear and contains dynamics, then the power factor can be expressed as a product of two terms, one resulting from the phase shift of the fundamental component of the current, and the other resulting from the current harmonics.

16.3.1 Linear Resistive Load, Nonsinusoidal Voltage

In this case, the current harmonics are in phase with, and proportional to, the voltage harmonics. As a result, all harmonics result in the net transfer of energy to the load. The current harmonic magnitudes and phases are

$$I_n = \frac{V_n}{R} \tag{16.16}$$

$$\theta_n = \varphi_n \qquad \text{so} \quad \cos(\theta_n - \varphi_n) = 1 \tag{16.17}$$

The rms voltage is again

$$(\text{rms voltage}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}} \tag{16.18}$$

and the rms current is

$$\text{(rms current)} = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}} = \sqrt{\frac{V_0^2}{R^2} + \sum_{n=1}^{\infty} \frac{V_n^2}{2R^2}} \\ = \frac{1}{R}\text{(rms voltage)} \tag{16.19}$$

By use of Eq. (16.6), the average power is

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\varphi_n - \theta_n) \\ = \frac{V_0^2}{R} + \sum_{n=1}^{\infty} \frac{V_n^2}{2R} \\ = \frac{1}{R}\text{(rms voltage)}^2 \tag{16.20}$$

Insertion of Eqs. (16.19) and (16.20) into Eq. (16.15) then shows that the power factor is unity. Thus, if the load is linear and purely resistive, then the power factor is unity regardless of the harmonic content of $v(t)$. The harmonic content of the load current waveform $i(t)$ is identical to that of $v(t)$, and all harmonics result in the transfer of energy to the load. This raises the possibility that one could construct a power distribution system based on nonsinusoidal waveforms in which the energy is efficiently transferred to the load.

16.3.2 Nonlinear Dynamical Load, Sinusoidal Voltage

If the voltage $v(t)$ contains a fundamental component but no dc component or harmonics, so that $V_0 = V_2 = V_3 = \ldots = 0$, then harmonics in $i(t)$ do not result in transmission of net energy to the load. The average power expression, Eq. (16.6), becomes

$$P_{av} = \frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_1) \tag{16.21}$$

However, the harmonics in $i(t)$ do affect the value of the rms current:

$$\text{(rms current)} = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}} \tag{16.22}$$

Hence, as in example 1 (Fig. 16.2), harmonics cause the load to draw more rms current from the source, but not more average power. Increasing the current harmonics does not cause more energy to be transferred to the load, but does cause additional losses in series resistive elements R_{series}.

Also, the presence of load dynamics and reactive elements, which causes the phase of the fundamental components of the voltage and current to differ $(\theta_1 - \varphi_1)$, also reduces the power factor. The $\cos(\varphi_1 - \theta_1)$ term in the average power Eq. (16.21) becomes less than unity. However, the rms value of the current, Eq. (16.22), does not depend on the phase. So shifting the phase of $i(t)$ with respect to $v(t)$ reduces the average power without changing the rms voltage or current, and hence the power factor is reduced.

By substituting Eqs. (16.21) and (16.22) into (16.15), we can express the power factor for the sinusoidal voltage in the following form:

$$(\text{power factor}) = \left(\frac{\dfrac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \displaystyle\sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) \left(\cos(\varphi_1 - \theta_1) \right) \tag{16.23}$$

$$= (\text{distortion factor})\ (\text{displacement factor})$$

So when the voltage contains no harmonics, then the power factor can be written as the product of two terms. The first, called the *distortion factor,* is the ratio of the rms fundamental component of the current to the total rms value of the current

$$(\text{distortion factor}) = \left(\frac{\dfrac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \displaystyle\sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) = \frac{(\text{rms fundamental current})}{(\text{rms current})} \tag{16.24}$$

The second term of Eq. (16.23) is called the *displacement factor,* and is the cosine of the angle between the fundamental components of the voltage and current waveforms.

The *Total Harmonic Distortion* (THD) is defined as the ratio of the rms value of the waveform not including the fundamental, to the rms fundamental magnitude. When no dc is present, this can be written:

$$(\text{THD}) = \frac{\sqrt{\displaystyle\sum_{n=2}^{\infty} I_n^2}}{I_1} \tag{16.25}$$

The total harmonic distortion and the distortion factor are closely related. Comparison of Eqs. (16.24) and (16.25), with $I_0 = 0$, eads to

$$(\text{distortion factor}) = \frac{1}{\sqrt{1 + (\text{THD})^2}} \tag{16.26}$$

This equation is plotted in Fig. 16.5. The distortion factor of a waveform with a moderate amount of distortion is quite close to unity. For example, if the waveform contains third harmonic whose magnitude is

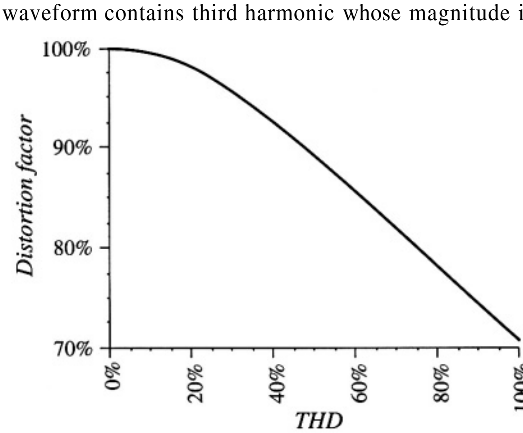

Fig. 16.5 Distortion factor vs. total harmonic distortion.

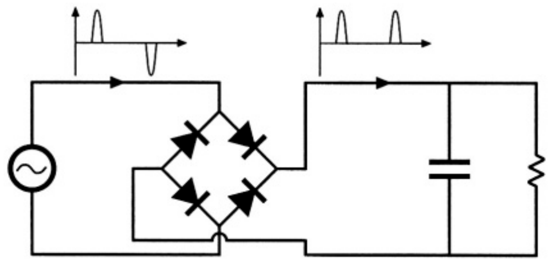

Fig. 16.6 Conventional peak detection rectifier.

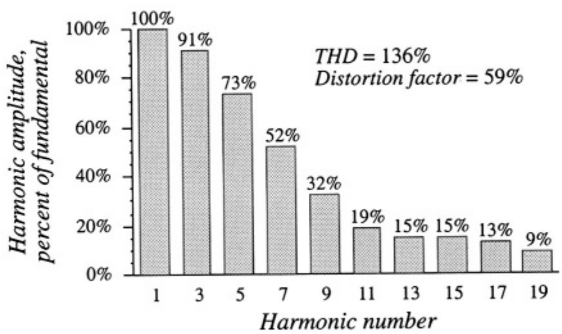

Fig. 16.7 Typical ac line current spectrum of a peak detection rectifier. Harmonics 1 to 19 are shown.

10% of the fundamental, the distortion factor is 99.5%. Increasing the third harmonic to 20% decreases the distortion factor to 98%, and a 33% harmonic magnitude yields a distortion factor of 95%. So the power factor is not significantly degraded by the presence of harmonics unless the harmonics are quite large in magnitude.

An example of a case in which the distortion factor is much less than unity is the conventional peak detection rectifier of Fig. 16.6. In this circuit, the ac line current consists of short-duration current pulses occurring at the peak of the voltage waveform. The fundamental component of the line current is essentially in phase with the voltage, and the displacement factor is close to unity. However, the low-order current harmonics are quite large, close in magnitude to that of the fundamental—a typical current spectrum is given in Fig. 16.7. The distortion factor of peak detection rectifiers is usually in the range 55% to 65%. The resulting power factor is similar in value.

In North America, the standard 120 V power outlet is protected by a 15 A circuit breaker. In consequence, the available load power quite limited. Derating the circuit breaker current by 20%, assuming typical efficiencies for the dc–dc converter and peak detection rectifier, and with a power factor of 55%, one obtains the following estimate for the maximum available dc load power:

$$\begin{aligned}&(\text{ac voltage})\ (\text{derated breaker current})\ (\text{power factor})\ (\text{rectifier efficiency})\\ &= (120\ \text{V})\quad (80\%\ \text{of}\ 15\ \text{A})\quad (0.55)\quad (0.98)\\ &= 776\ \text{W}\end{aligned} \tag{16.27}$$

The less-than-unity efficiency of a dc–dc converter would further reduce the available dc load power. Using a peak detection rectifier to supply a load power greater than this requires that the user install higher amperage and/or higher voltage service, which is inconvenient and costly. The use of a rectifier circuit having nearly unity power factor would allow a significant increase in available dc load power:

$$\begin{aligned}&\text{(ac voltage) (derated breaker current) (power factor) (rectifier efficiency)}\\&= (120\text{ V}) \quad (80\%\text{ of }15\text{ A}) \quad (0.99) \quad (0.93)\\&= 1325\text{ W}\end{aligned} \tag{16.28}$$

or almost twice the available power of the peak detection rectifier. This alone can be a compelling reason to employ high quality rectifiers in commercial systems.

16.4 POWER PHASORS IN SINUSOIDAL SYSTEMS

The apparent power is defined as the product of the rms voltage and rms current. Apparent power is easily measured—it is simply the product of the readings of a voltmeter and ammeter placed in the circuit at the given surface. Many power system elements, such as transformers, must be rated according to the apparent power that they are able to supply. The unit of apparent power is the volt-ampere, or VA. The power factor, defined in Eq. (16.15), is the ratio of average power to apparent power.

In the case of sinusoidal voltage and current waveforms, we can additionally define the *complex power* $\boldsymbol{S}$ and the *reactive power* $\boldsymbol{Q}$. If the sinusoidal voltage $v(t)$ and current $i(t)$ can be represented by the phasors $\boldsymbol{V}$ and $\boldsymbol{I}$, then the complex power is a phasor defined as

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = P + jQ \tag{16.29}$$

Here, $\boldsymbol{I}^*$ is the complex conjugate of $\boldsymbol{I}$, and j is the square root of –1. The magnitude of $\boldsymbol{S}$, or $\|\boldsymbol{S}\|$, is equal to the apparent power, measured in VA. The real part of $\boldsymbol{S}$ is the average power P, having units of watts. The imaginary part of $\boldsymbol{S}$ is the reactive power Q, having units of reactive volt-amperes, or VARs.

A phasor diagram illustrating $\boldsymbol{S}$, P, and Q, is given in Fig. 16.8. The angle $(\varphi_1 - \theta_1)$ is the angle between the voltage phasor $\boldsymbol{V}$ and the current phasor $\boldsymbol{I}$. $(\varphi_1 - \theta_1)$ is additionally the phase of the complex power $\boldsymbol{S}$. The power factor in the purely sinusoidal case is therefore

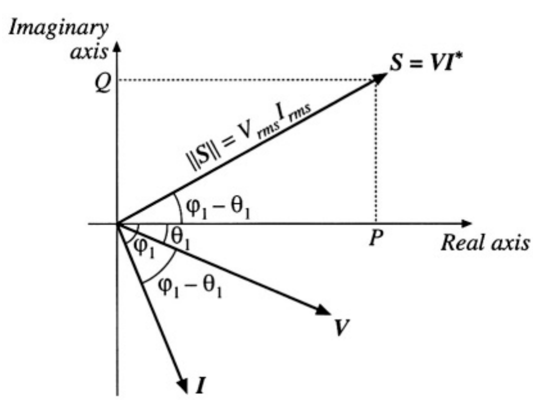

Fig. 16.8 Power phasor diagram, for a sinusoidal system, illustrating the voltage, current, and complex power phasors.

$$\text{power factor} = \frac{P}{|S|} = \cos\left(\varphi_1 - \theta_1\right) \tag{16.30}$$

It should be emphasized that this equation is valid only for systems in which the voltage and current are purely sinusoidal. The distortion factor of Eq. (16.24) then becomes unity, and the power factor is equal to the displacement factor as in Eq. (16.30).

The reactive power Q does not lead to net transmission of energy between the source and load. When reactive power is present, the rms current and apparent power are greater than the minimum amount necessary to transmit the average power P. In an inductor, the current lags the voltage by 90°, causing the displacement factor to be zero. The alternate storing and releasing of energy in an inductor leads to current flow and nonzero apparent power, but the average power P is zero. Just as resistors consume real (average) power P, inductors can be viewed as consumers of reactive power Q. In a capacitor, the current leads to voltage by 90°, again causing the displacement factor to be zero. Capacitors supply reactive power Q, and are commonly placed in the utility power distribution system near inductive loads. If the reactive power supplied by the capacitor is equal to the reactive power consumed by the inductor, then the net current (flowing from the source into the capacitor-inductive-load combination) will be in phase with the voltage, leading unity power factor and minimum rms current magnitude.

It will be seen in the next chapter that phase-controlled rectifiers produce a nonsinusoidal current waveform whose fundamental component lags the voltage. This lagging current does not arise from energy storage, but it does nonetheless lead to a reduced displacement factor, and to rms current and apparent power that are greater than the minimum amount necessary to transmit the average power.

16.5 HARMONIC CURRENTS IN THREE PHASE SYSTEMS

The presence of harmonic currents can also lead to some special problems in three-phase systems. In a four-wire three-phase system, harmonic currents can lead to large currents in the neutral conductors, which may easily exceed the conductor rms current rating. Power factor correction capacitors may experience significantly increased rms currents, causing them to fail. In this section, these problems are examined, and the properties of harmonic current flow in three-phase systems are derived.

16.5.1 Harmonic Currents in Three-Phase Four-Wire Networks

Let us consider the three-phase four-wire network of Fig. 16.9. In general, we can express the Fourier series of the line currents and line-neutral voltages as follows:

$$\begin{aligned}
i_a(t) &= I_{a0} + \sum_{k=1}^{\infty} I_{ak} \cos\left(k\omega t - \theta_{ak}\right) \\
i_b(t) &= I_{b0} + \sum_{k=1}^{\infty} I_{bk} \cos\left(k(\omega t - 120^\circ) - \theta_{bk}\right) \\
i_c(t) &= I_{c0} + \sum_{k=1}^{\infty} I_{ck} \cos\left(k(\omega t + 120^\circ) - \theta_{ck}\right)
\end{aligned} \tag{16.31}$$

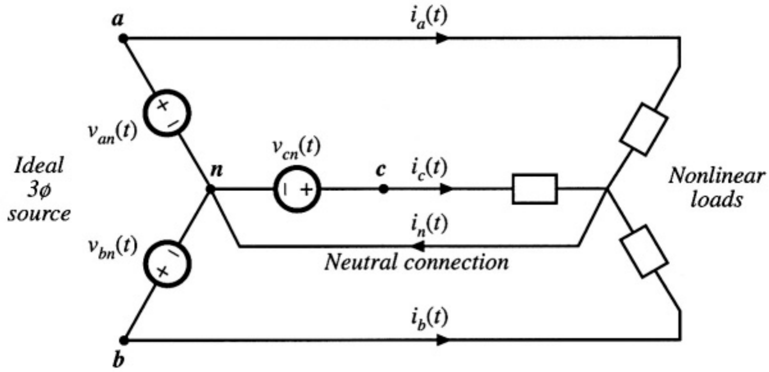

Fig. 16.9 Current flow in a three-phase four-wire network.

$$
\begin{aligned}
v_{an}(t) &= V_m \cos(\omega t) \\
v_{bn}(t) &= V_m \cos(\omega t - 120^\circ) \\
v_{cn}(t) &= V_m \cos(\omega t + 120^\circ)
\end{aligned}
\tag{16.32}
$$

The neutral current is therefore $i_n = i_a + i_b + i_c$, or

$$
\begin{aligned}
&i_n(t) = I_{a0} + I_{b0} + I_{c0} + \\
&\sum_{k=1}^{\infty} \left[I_{ak}\cos(k\omega t - \theta_{ak}) + I_{bk}\cos(k(\omega t - 120^\circ) - \theta_{bk}) + I_{ck}\cos(k(\omega t + 120^\circ) - \theta_{ck}) \right]
\end{aligned}
\tag{16.33}
$$

When the load is unbalanced (even though the voltages are balanced and undistorted), we can say little else about the neutral and line currents. If the load is unbalanced and nonlinear, then the line and neutral currents may contain harmonics of any order, including even and triplen harmonics.

Equation (16.33) is considerably simplified in the case where the loads are balanced. A balanced nonlinear load is one in which $I_{ak} = I_{bk} = I_{ck} = I_k$ and $\theta_{ak} = \theta_{bk} = \theta_{ck} = \theta_k$, for all k; that is, the harmonics of the three phases all have equal amplitudes and phase shifts. In this case, Eq. (16.33) reduces to

$$
i_n(t) = 3I_0 + \sum_{k=3,6,9,\cdots}^{\infty} 3I_k \cos(k\omega t - \theta_k)
\tag{16.34}
$$

Hence, the fundamental and most of the harmonics cancel out, and do not appear in the neutral conductor. Thus, it is in the interests of the utility to balance their nonlinear loads as well as their harmonics.

But not all of the harmonics cancel out of Eq. (16.34): the dc and *triplen* (triple-n, or 3,6,9,...) harmonics add rather than cancel. The rms neutral current is

$$
i_{n,rms} = 3\sqrt{I_0^2 + \sum_{k=3,6,9,\cdots}^{\infty} \frac{I_k^2}{2}}
\tag{16.35}
$$

Example

A balanced nonlinear load produces line currents containing fundamental and 20% third harmonic: $i_{an}(t) = I_1\cos(\omega t - \theta_1) + 0.2I_1\cos(3\omega t - \theta_3)$. Find the rms neutral current, and compare its amplitude to the rms line current amplitude.

Solution:

$$i_{n,\,rms} = 3\sqrt{\frac{(0.2I_1)^2}{2}} = \frac{0.6\,I_1}{\sqrt{2}}$$
$$i_{1,\,rms} = \sqrt{\frac{I_1^2 + (0.2I_1)^2}{2}} = \frac{I_1}{\sqrt{2}}\sqrt{1+0.04} \approx \frac{I_1}{\sqrt{2}} \tag{16.36}$$

So the neutral current magnitude is 60% of the line current magnitude! The triplen harmonics in the three phases add, such that 20% third harmonic leads to 60% third harmonic neutral current. Yet the presence of the third harmonic has very little effect on the rms value of the line current. Significant unexpected neutral current flows.

16.5.2 Harmonic Currents in Three-Phase Three-Wire Networks

If there is no neutral connection to the wye-connected load, as in Fig. 16.10, then $i_n(t)$ must be zero. If the load is balanced, then Eq. (16.34) still applies, and therefore the dc and triplen harmonics of the load currents must be zero. Therefore, the line currents i_a, i_b, and i_c cannot contain triplen or dc harmonics. What happens is that a voltage is induced at the load neutral point n', containing dc and triplen harmonics, which eliminates the triplen and dc load current harmonics.

This result is true only when the load is balanced. With an unbalanced load, all harmonics can appear in the line currents, including triplen and dc. In practice, the load is never exactly balanced, and some small amounts of third harmonic line currents are measured.

With a delta-connected load as in Fig. 16.11, there is also no neutral connection, so the line currents cannot contain triplen or dc components. But the loads are connected line-to-line, and are excited by undistorted sinusoidal voltages. Hence triplen harmonic and dc currents do, in general, flow through the nonlinear loads. Therefore, these currents simply circulate around the delta. If the load is balanced, then again no triplen harmonics appear in the line currents.

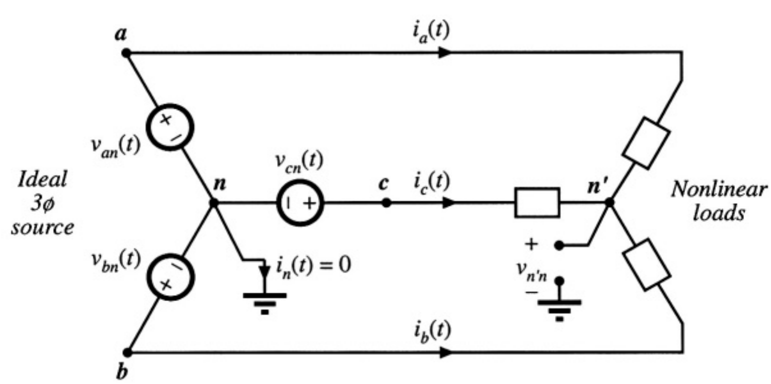

Fig. 16.10 Current flow in a three-phase three-wire wye-connected network.

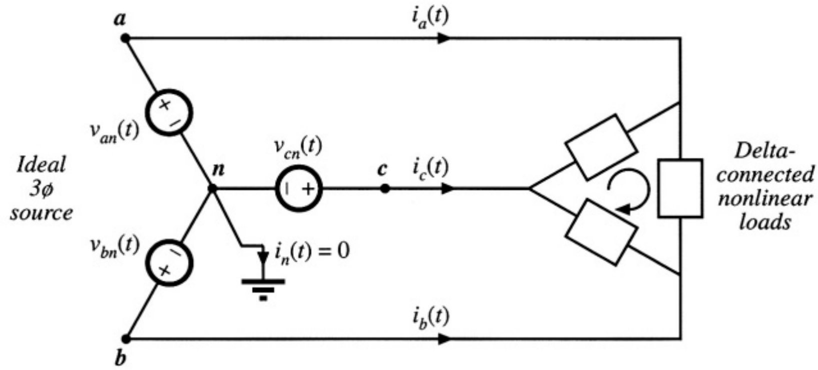

Fig. 16.11 A balanced nonlinear delta-connected load may generate triplen current harmonics. These harmonics circulate around the delta, but do not flow through the lines if the load is balanced.

16.5.3 Harmonic Current Flow in Power Factor Correction Capacitors

Harmonic currents tend to flow through shunt-connected power factor correction capacitors. To some extent, this is a good thing because the capacitors tend to low-pass filter the power system currents, and prevent nonlinear loads from polluting the entire power system. The flow of harmonic currents is then confined to the nonlinear load and local power factor correction capacitors, and voltage waveform distortion is reduced. High-frequency harmonic currents tend to flow through shunt capacitors because the capacitor impedance decreases with frequency, while the inductive impedance of transmission lines increases with frequency. In this sense, power factor correction capacitors mitigate the effects of harmonic currents arising from nonlinear loads in much the same way that they mitigate the effects of reactive currents that arise from inductive loads.

But the problem is that the power factor correction capacitors may not be rated to handle these harmonic currents, and hence there is a danger that the capacitors may overheat and fail when they are exposed to significant harmonic currents. The loss in capacitors is modeled using an *equivalent series resistance* (esr) as shown in Fig. 16.12. The esr models dielectric loss (hysteresis of the dielectric *D–E* loop), contact resistance, and foil and lead resistances. Power loss occurs, equal to $i_{rms}^2(esr)$. Dielectric materials are typically poor conductors of heat, so a moderate amount of power loss can cause a large temperature rise in the center of the capacitor. In consequence, the rms current must be limited to a safe value.

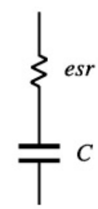

Fig. 16.12 Capacitor equivalent circuit. Losses are modeled by an equivalent series resistance (esr).

Typical power factor correction capacitors are rated by voltage V, frequency f, and reactive power in kVARs. These ratings are computed from the capacitance C and safe rms current I_{rms}, assuming undistorted sinusoidal waveforms, as follows:

$$\text{rated rms voltage } V_{rms} = \frac{I_{rms}}{2\pi f C} \tag{16.37}$$

$$\text{rated reactive power} = \frac{I_{rms}^2}{2\pi f C} \tag{16.38}$$

In an undistorted system, the rms current, and hence also the capacitor esr loss, cannot increase unless the rms voltage is also increased. But high-frequency harmonics can lead to larger rms currents without an increased voltage. Any harmonics that flow result in increased rms current beyond the expected value predicted by Eq. (16.37). If the capacitor is not rated to handle additional power loss, then failure or premature aging can occur.

16.6 AC LINE CURRENT HARMONIC STANDARDS

Besides the increased currents and reduced power factors of peak detection rectifiers, the harmonics themselves can be detrimental: if large enough in magnitude, they can pollute the power system. Harmonic currents cause distortion of the voltage waveform via the power system series impedance. These voltage harmonics can interfere with the operation of nearby loads. As noted previously, increased currents in shunt capacitors, and increased losses in distribution transformers and ac machines, can lead to premature aging and failure of these devices. Odd triplen harmonics (triple-*n*: 3^{rd}, 9^{th}, 15^{th}, etc.) lead to unexpectedly large neutral currents in three-phase systems. Harmonic currents can also excite system resonances some distance from their source, with results that are difficult to predict. For these reasons, a number of organizations have adopted standards that limit the magnitudes of the harmonic currents that a load is allowed to inject into the ac line. The US military was one of the early organizations to recognize these problems; the very strict 3% limit was initially adopted. The standards adopted by the IEC and IEEE are more recent, and are intended for conventional utility systems. A fourth example, not discussed here, is the telephone interference factor, which limits power distribution system harmonics in cases when telephone lines and power lines share the same poles.

16.6.1 International Electrotechnical Commission Standard 1000

This international agency adopted a first draft of their IEC 555 standard in 1982. It has since undergone a number of revisions, and has been superceded by IEC 1000 [7]. This standard is now enforced in Europe, making it a de facto standard for commercial equipment intended to be sold worldwide.

The IEC 1000-3-2 standard covers a number of different types of low power equipment, with differing harmonic limits. It specifically limits harmonics for equipment having an input current of up to 16 A, connected to 50 or 60 Hz, 220 V to 240 V single phase circuits (two or three wire), as well as 380 V to 415 V three phase (three or four wire) circuits. In a city environment such as a large building, a large fraction of the total power system load can be nonlinear. For example, a major portion of the electrical load in a building is comprised of fluorescent lights, which present a very nonlinear characteristic to the utility system. A modern office may also contain a large number of personal computers, printers, copiers, etc., each of which may employ peak detection rectifiers. Although each individual load is a negligible fraction of the total local load, these loads can collectively become significant.

The IEC 1000-3-2 standard defines several categories of equipment, each of which is covered by a different set of harmonic limits. As an example, Table 16.1 shows the harmonic limits for *Class A* equipment, which includes low harmonic rectifiers for computer and other office equipment.

The European norm EN 61000-3-2 defines similar limits.

Table 16.1 IEC 1000-3-2 Harmonic current limits, class A

Odd harmonics		Even harmonics	
Harmonic number	Maximum current	Harmonic number	Maximum current
3	2.30 A	2	1.08 A
5	1.14 A	4	0.43 A
7	0.77 A	6	0.30 A
9	0.40 A	$8 \le n \le 40$	$0.23\ \text{A} \cdot (8/n)$
11	0.33 A		
13	0.21 A		
$15 \le n \le 39$	$0.15\ \text{A} \cdot (15/n)$		

16.6.2 IEEE/ANSI Standard 519

In 1993, the IEEE published a revised draft standard limiting the amplitudes of current harmonics, *IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters.* The harmonic limits are based on the ratio of the fundamental component of the load current I_L to the short circuit current at the point of common coupling (PCC) at the utility I_{sc}. Stricter limits are imposed on large loads than on small loads. The limits are similar in magnitude to IEC 1000, and cover high voltage loads (of much higher power) not addressed by IEC 1000. Enforcement of this standard is presently up to the local utility company.

The odd harmonic limits for general distribution systems at voltages of 120 V to 69 kV are listed in Table 16.2. The limits for even harmonics are 25% of the odd harmonic limits. Limits for general distribution systems at 69.001 kV to 161 kV are 50% of the values listed in Table 16.2. DC current components and half-wave rectifiers are not allowed.

It is the responsibility of the power consumer to meet these current harmonic standards. Standard IEEE-519 also specifies maximum allowable voltage harmonics, listed in Table 16.3. It is the responsibility of the utility, or power supplier, to meet these limits. Both total harmonic distortion and maximum individual harmonic magnitudes are limited.

Table 16.2 IEEE-519 Maximum odd harmonic current limits for general distribution systems, 120 V to 69 kV

I_{sc}/I_L	$n < 11$	$11 \le n < 17$	$17 \le n < 23$	$23 \le n < 35$	$35 \le n$	THD
< 20	4.0%	2.0%	1.5%	0.6%	0.3%	5.0%
20-50	7.0%	3.5%	2.5%	1.0%	0.5%	8.0%
50-100	10.0%	4.5%	4.0%	1.5%	0.7%	12.0%
100-1000	12.0%	5.5%	5.0%	2.0%	1.0%	15.0%
> 1000	15.0%	7.0%	6.0%	2.5%	1.4%	20.0%

Table 16.3 IEEE-519 Voltage distortion limits

Bus voltage at PCC	Individual harmonics	THD
69 kV and lower	3.0%	5.0%
69.001 kV to 161 kV	1.5%	2.5%
Above 161 kV	1.0%	1.5%

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PROBLEMS

16.1 Passive rectifier circuit. In the passive rectifier circuit of Fig. 16.13, L is very large, such that the inductor current $i(t)$ is essentially dc. All components are ideal.

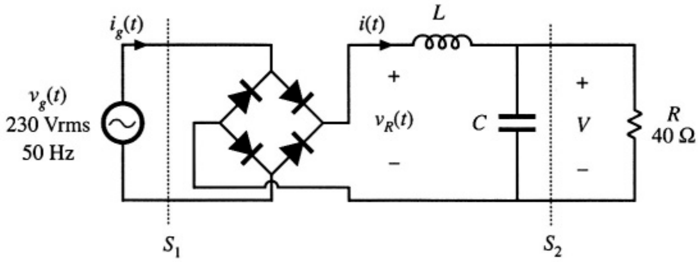

Fig. 16.13 Passive rectifier circuit of Problem 16.1

(a) Determine the dc output voltage, current, and power.

(b) Sketch the ac line current waveform $i_g(t)$ and the rectifier output voltage waveform $v_R(t)$.

(c) Determine the ac line current rms magnitude, fundamental rms magnitude, and third harmonic rms magnitude. Does this rectifier network conform to the IEC-1000 harmonic current limits?

(d) Determine the power factor, measured at surfaces S_1 and S_2.

16.2 The three-phase rectifier of Fig. 16.14 is connected to a balanced 60 Hz 3øac 480 V (rms, line-line) sinusoidal source as shown. All elements are ideal. The inductance L is large, such that the current $i(t)$ is essentially constant, with negligible 360 Hz ripple.

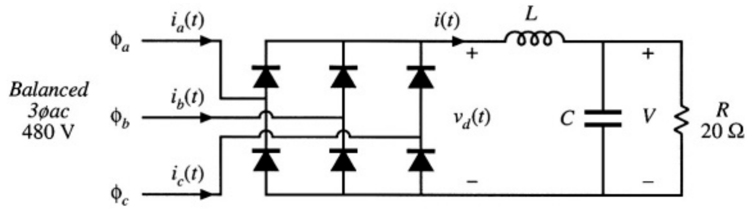

Fig. 16.14 Three-phase rectifier circuit of Problem 16.2

(a) Sketch the waveform $v_d(t)$

(b) Determine the dc output voltage V.

(c) Sketch the line current waveforms $i_a(t)$, $i_b(t)$, and $i_c(t)$.

(d) Find the Fourier series of $i_a(t)$

(e) Find the distortion factor, displacement factor, power factor, and line current THD.

16.3 Harmonic pollution police. In the network of Fig. 16.15, voltage harmonics are observed at the indicated surface. The object of this problem is to decide whether to blame the source or the load for the observed harmonic pollution. Either the source element or the load element contains a nonlinearity that generates harmonics, while the other element is linear.

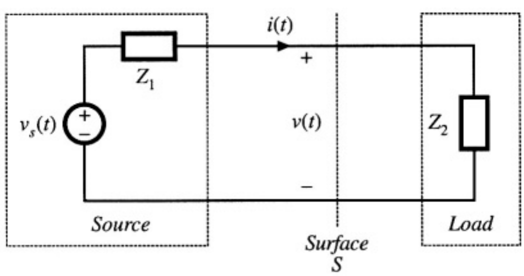

Fig. 16.15 Single-phase power system of Problems 16.3 to 16.5

(a) Consider first the case where the load is a passive linear impedance $Z_2(s)$, and hence its phase lies in the range $-90° \le \angle Z_2(j\omega) \le +90°$ for all positive ω. The source generates harmonics. Express the average power P in the form

$$P = \sum_{n=0}^{\infty} P_n$$

where P_n is the average power transmitted to the load by harmonic number n. What can you say about the polarities of the P_ns?

(b) Consider next the case where the load is nonlinear, while the source is linear and can be modeled by a Thevenin-equivalent sinusoidal voltage source and linear impedance $Z_1(s)$. Again express the average power P as a sum of average powers, as in part (a). What can you say about the polarities of the P_ns in this case?

(c) The following Fourier series are measured:

Harmonic number	$v(t)$		$i(t)$	
	Magnitude	Phase	Magnitude	Phase
1	230 V	0°	6 A	– 20°
3	20 V	180°	4 A	20°
5	8 V	60°	1 A	– 110°

Who do you accuse? Explain your reasoning.

16.4 For the network and waveforms of Problem 16.3, determine the power factor at the indicated surface, and the average power flowing to the load. Harmonics higher in frequency than the fifth harmonic are negligible in magnitude.

16.5 Repeat Problem 16.3(c), using the following Fourier series:

Harmonic number	$v(t)$		$i(t)$	
	Magnitude	Phase	Magnitude	Phase
1	120 V	0°	5 A	25°
3	4 V	60°	0.5 A	40°
5	2 V	– 160°	0.2 A	– 100°

16.6 A balanced three-phase wye-connected load is constructed using a 20 Ω resistor in each phase. This load is connected to a balanced three-phase wye-connected voltage source, whose fundamental voltage component is 380 Vrms line-to-line. In addition, each (line-to-neutral) voltage source produces third and fifth harmonics. Each harmonic has amplitude 20 Vrms, and is in phase with the (line-to-neutral) fundamental.

(a) The source and load neutral points are connected, such that a four-wire system is obtained. Find the Fourier series of the line currents and the neutral current.

(b) The neutral connection is broken, such that a three-wire system is obtained. Find the Fourier series of the line currents. Also find the Fourier series of the voltage between the source and load neutral points.